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KANT'S FIRST ANTINOMY.[1]

PERHAPS no one would take quite seriously, argument by argument, Kant's presentation of the "first antinomy." It may not, that is, have been made out impossible to think of a world infinite in extent, equally impossible to think of one that is finite; forbidden to conceive of a world whose past history has no beginning, no less forbidden to conceive of one whose past history has a first moment.

Yet out of the 'much argument' there may be seen to emerge a puzzle that is not without suggestion. It is this: There is no method by which we may determine the spread of bodies in space, the history of their behavior in time, save that of bare observation. There are, however, the most plausible reasons for asserting that the observations indispensable to this determining form an endless series. If now there is but one method of settling an issue of fact, and if *that* method will not settle it, are we not face to face with the most puzzling of conceptions, *an unknowable fact?*

One naturally thinks first of establishing the reality of the issue. *Is* an infinite series of experiments, one asks, indispensable to the answering of these two questions of extent and duration? Here, many reflections occur to a modern that have received no consideration at the hands of Kant. Those connected with the problem of the space distribution of bodies are certainly the less vital, yet they should be set forth in some manner. Thus, it may occur to one that Kant has made an assumption requiring analysis when he thinks of the method by which we determine the existence of bodies at a certain distance from the earth (say) as involving a consumption of time proportionate to the distance. Certainly if the only method were to send a messenger to such regions, or to await a messenger (say, of light) from them, then, however rapidly the messenger were to travel, a measurable time would be required for him to arrive and report. And as the journey must be infinite in order to exhaust the infinite reaches of space, the time involved would be infinite no less. But the consideration thrusts itself upon us at this point that not all physical influences are known to be propagated. The gravitational effect of one mass upon another is, so far as experiment can decide, instantaneous. If so, it is conceivable that the presence of masses at all distances from a given finite system would be reflected in the momentary behavior of this system. In order that such a state of affairs may result, however, the law of the force which one body exerts on another must be such that no two distributions should give the same map of lines of force in a given finite region. But so far as gravitation is concerned, there are at least certain 'critical cases' in which more than one, in fact an infinite number of distributions of mass would give exactly the same force-map for a circumscribed region. One such case would be that in which the region in question lay inside of a spherical shell. The intensity in the interior of such a shell being zero, it follows that the presence of the shell would not be revealed by any momentary behavior of systems lying within it. To assure ourselves of its presence or absence, we should require some other method than that of observing the gravitational behavior of a finite system of bodies.[1]

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[1] Read before the Baltimore meeting of the American Philosophical Association, December, 1908.

If then no law of the interaction of bodies is empirically known which would enable us in a finite number of experiments to determine the distribution of bodies throughout space by examining the behavior of bodies within a finite region, we can have no quarrel with Kant's results. Whether bodies are distributed throughout the infinity of space or are confined to a region of it remains an experimentally unanswerable question.

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[1] To be sure, the theorem establishing the zero intensity of gravitational force in the interior of a spherical shell depends upon the assumption of the law of the inverse square. To establish the theorem that *no* law of nature can be defined which reflects distribution throughout an infinite region in the behavior of bodies in a finite region requires two steps. First, a law must be conceived that yields no such critical cases as that defined by the law of gravitation. Second, such a law must give a force-function that lies at a finite distance from any given force-function which yields such critical cases. Otherwise viewed, the necessity of a probable error of observation, an infinite number of observations would be required to decide between the function that does and the function that does *not* reflect an infinite distribution in a finite one. I am content to leave so delicate a question of mathematics to future treatments of the problem. The difficulties of the discussion seem to me out of proportion to the importance of the issue.

But when we turn from the problem of distribution in space to that of history in time, the facile analogy by which Kant himself is wont to pass from the one question to the other seems something too unconsidered. For we have undoubtedly the vague suspicion that the history of the past is embodied in the *now* in a much more complete way than the structure of the *yonder* is reflected in the *here*.

I am not inclined to stick on the point that the history of the past is a present document,—is, as Hume would say, a lively idea associated with a present impression. I am rather tempted to put the question whether or not the history, however recorded, of a system is indispensable to a prediction of its future conduct.

When Kant suggests that past history throws its light upon present conditions (a light indispensable to our vision into the future) he is proposing no new idea, nor one likely to offend the commonest of common-sense. Only, the sciences in which retrospect is necessary to prevision are the relatively superficial sciences,—biology, sociology, psychology. The adjective "superficial" is not of course meant to be abusive. It is obvious that certain sciences owe their claim upon our attention to their success in inventing types of explanation that are applicable when the data for applying more thorough-going sciences are wanting. For example, few would be prepared to maintain that the training of an organism left no trace on the intimate anatomy of that organism, although it is true no such trace has been observed. We have not advanced beyond the stage in which the anatomical changes effected by training are referred to in figures borrowed from other sciences; we still confine ourselves to speaking of the well-worn paths of association and the like. Such expressions only give name to a hope and a faith, but they do at least testify to that much,—to a faith, namely, that did we know enough of minute structural differences we should be able to predict a difference in the reactions of two organisms to the same stimulus when these organisms, as we now view them, are unlike only in training. Not knowing so much in the way of structural detail, we must include among the data indispensable to prediction a knowledge of the past history of the organism, and we introduce among the influences of which account must be taken in explaining behavior 'the force of habit,' 'the direction of memory,' and like expressions which in their very wording mimic the categories of mechanical explanation. To have invented such devices for a ready prediction where the data required by more general, and so deeper, sciences are lacking, is just the contribution of those sciences that I have called superficial. But we must recognize that such sciences are conscious of their superficial, make-shift character, that they have themselves the ambition to dispense with their special devices, and their first step in this sense would be to eliminate if possible an appeal to past history as explanatory.

We cannot say, of course, that any given scientific ideal is bound to be realized by a complacent world of fact. It is perhaps true that biology has made some progress in its effort to trace the structural modifications wrought in an organism by its experience. On the other hand, an increasing knowledge of the inanimate world has revealed, where we least expected to find them, classes of phenomena in which account has to be taken of past history. One which has suggested more frequently than any other that puzzling phrase 'the memory of matter,' is the well-known phenomenon of 'hysteresis.' It would appear, *e. g.*, that two magnets in exactly the same present condition would react differently to the same treatment if they had come into that condition by different ways. In other words, we have to know the past history of a magnet, just as we do of an organism, if we are able to predict its future behavior. Of course if instances of this kind should multiply in the inorganic world, and if in the organic world science should ultimately give up its hope of dispensing with the kind of explanation we have called superficial, the *now* would no more embody the *then* than the *here* reflects the *yonder*, and Kant's transition from the problem of space distribution to that of past history would be beyond criticism. But as just the opposite is the case, as in the inorganic world the situations in which we have to take into account the path by which a system comes into a given condition are so rare as to excite wonder, and as in the organic world reduction rather than extension is the result of study, it is no mere idle curiosity, but a pressing requirement of sound method to reconsider Kant's treatment of history from the point of view of an image of nature which regards that history as sufficiently recounted if we substitute negative values of time in a formula now observed to hold throughout the known world, and solve for all the dependent variables.

While such an assumption vastly simplifies our conception of natural science, it complicates or rather subtilizes beyond expectation the particular problem with which we are here dealing. Questions not easy to answer begin to crowd upon us. What sort of a formula would mean an infinite past history? What sort a finite? What kind of experiment would be necessary and sufficient to decide between the two formulas? Would this experiment involve a finite or an infinite series of observations?

The first question is not difficult to answer, so little difficult indeed that a formula which means an infinite past and an infinite future history is the one which every scientist carries around with him as the more or less conscious starting-point of his reflections on natural philosophy. Arrhenius, for example, does not hesitate to say that no other world-view is to him conceivable than such as makes past history infinite. All such formulas have at least this in common. They succeed in expressing everything that varies in nature, — whether positions in space, electric charges, magnetic poles, — as functions of a single variable called time; so that for all real values of this independent variable we obtain real values for at least one kind of the dependent variables (*e. g.*, position) and values greater than zero for the time-rate of change of at least one kind of these variables (*e. g.*, velocity). A system of gravitating masses with velocities normal to the lines connecting the masses would be a system with an infinite past, and we might superimpose upon such a system various phenomena of radiation which would not preclude such infinity.

Neither would the attempt to construct an image of a system with a finite past or future history seem to offer insuperable difficulty. Perhaps it will lie nearer to our experience to think first of a system destined to run down in a finite time. To this end nothing more seems required than to replace the *Wärmetod*, which Clausius conceived to be the future actually awaiting the system we know. Suppose in a gravitating system we imagine the masses to behave as though moving through a resisting medium. One effect would be constantly to diminish the velocities normal to the radii vectores of the system, velocities now conceived to remain constant. The consequence would be a gradual approach along a spiral path of satellites toward planets, of planets toward suns, of suns toward each other. If the following of such paths resulted in impact with velocities greater than zero, even if we assumed the masses to be perfectly elastic, the conditions would require that the kinetic energy of the masses of the system should gradually diminish, while such gain in potential energy as could be established would not be sufficient to make up the loss; so that the whole system would tend toward a zero of energy, *i. e.*, toward a state of equilibrium. To be sure, so far as experience acquaints us with systems of masses moving through resisting mediums, we look to phenomena of the medium, such as convection currents and radiation effects, to keep the total of energy constant. But nothing assures us so long as there is an error of observation that the sum will be made up, and if it be not, one form of change does not completely replace another; the whole history of the system, which is precisely a record of its changes, tends toward an end.

There remains a point to be considered, or rather two points. A system tending toward equilibrium may not so approach the limit as to attain to it in a finite time. And, in the second place, since we are inspired to these reflections by Kant's treatment of the antinomies, we are without interest in images constructed in terms of laws of force which a finite number of experiments could prove to be untrue to fact. But I am content to treat these two problems in geometrical form. If we constructed a graph of the energy of a system in which this quantity remained constant, laying off time along the X axis, and the corresponding energy along the Y, the energy would be represented by a line parallel to the X axis. The fact that there is always an experimental error makes it, however, impossible to distinguish between this parallel and a second straight line which would intersect the X axis at a point representing the time when the energy of the system had become zero. As this state of affairs remains unchanged so long as the probable error is greater than zero, and as the very formula by means of which we express the probable error of experimental results implies that nothing short of an infinite series of experiments would enable us to reduce such an error to zero, it follows that no finite series would enable us to include the possibility of a *finite* future history. On the other hand it might be that without going outside of the types of curves which we are acustomed to regard as alone capable of expressing physical laws, we could find none lying within the limits of probable error which did not intersect the axis of X. We should still not have excluded the possibility of an *infinite* future. For unless the forces we can ascribe to the system are very different from any with which we are acquainted, the energy of the system must increase with the separation of its parts in space. But we have already accepted Kant's result that no finite series of experiments could determine this distribution for us. Consequently, though we were able to establish a law which would require any *finite* system to run down in a finite time, we could not establish the finitude of the system we call the world.

I have presented the problem of a finite future first, in deference to our prejudices. Conceptually, however, there is no greater difficulty in defining the conditions of a finite past. The transition from the one to the other involves nothing more complicated than a change of sign, the substitution of an accelerating medium for a retarding. It is true that experience makes the latter concept less familiar to us, but it can render it no more difficult to define. To go forward to an end and to go backward to a beginning are problems of the same order.

Yet there are certain objections that may well present themselves to the mind of one attempting to follow in imagination this finite history to its beginning or end. To one of these our attention has again and again been called in the history of reflection. Kant himself urges it as conclusive against the hypothesis of a finite past. If, namely, we conceive our system as starting from a condition of equilibrium, what in its previous history of rest decides the moment at which its slumber shall be broken, its balance upset?

The question is a delicate one, and yet I am tempted to maintain that its only answer lies in the ungracious suggestion that it ought not to have been asked. That it is asked is due to our forgetfulness of a simple caution respecting the meaning of time. Our need to deal experimentally with fragmentary bits of nature, systems set up in our laboratories or such as nature has fairly well isolated for us, has left us with the mental habit of looking upon the coördinates of position together with the clock that marks time as furnished for us *from without the system.* When, however, we pass from bits of nature to nature as a whole, we realize that our coördinates of position and our clock are parts of the thing we are dealing with. We can record only distances between masses, between charges, etc., and watch the way in which these distances change with respect to each other. Our clock then becomes a changing coördinate of our system. To take the path of some member of this system as our clock, *defines* those changes to be taking place at a constant rate, whose increments are proportional to the increments of path laid off by the body we have chosen. Now, obviously, the choice of such a clock is quite arbitrary; and this arbitrary element disappears, the issue between a finite and an infinite history for a given system is experimentally settled, only when we have made trial of *every possible clock.* I mean that to think of a system with a finite history, it is necessary to think of each possible clock as having run down in terms of each other, it is not enough to think of the system as having run down in terms of *a* clock. Suppose, namely, that we had arrived at the point of recognizing that only one coördinate of the system is changing, *i. e.*, as we substitute in our equations successive increments of this coördinate, the rate of change of all others with respect to it is found to be zero, its rate of change with respect to them consequently infinite. Our task of tracing the history of the system in terms of its last possible intrinsic clock is not completed, in a sense that would lead us to pronounce this history finite, until we come to a last substitution such that on trying it we find that all the coördinates of our system become imaginary. For this result we should be forced to interpret to mean that the coördinate in question could not take on the value substituted, that here the last possible clock stops, that the system as a whole has come to rest.

And just at this point, if we imagine our substitutions to have been toward a beginning, our questioner would ask: What happened prior to this? Is there not an infinite history of rest to be added on to a finite history of change? I answer: A part of a system may have a history of rest in terms of a clock found in another part of the system not at rest; but there is no sense in which we can speak of a history of rest in connection with a whole system. If then one asks: What marks the moment when the static system begins to move? Why one moment rather than another? The only answer is: There are no moments for a static system, hence no "one rather than another." We have been faced with one of those "*ungereimte Fragen*" that Kant is sharp to criticise when he detects the "*Ungereimtheit*," but is himself capable of puzzling over when the trick has escaped him.

But there is one other difficulty that suggests itself. If it is maintained that an infinite number of observations is indispensable to the establishing of the fact that the future history of the world is but finite, whence, in this case, is to come the time in which to perform the experiments involved? Would not the experience that we could not perform the series be itself a finite experiment, or less subtly stated, must not the experience which corresponds to the establishing of a last moment, an end of things, lie within finite range in case the facts are such as to confirm the hypothesis of such an end?

One shrinks before the subtlety of the only answer which seems consistent. The end of experience is not the experience of an end. Should the conditions which make experience possible themselves vanish, I see no sense in which this vanishing would itself be a fact open to observation. Let him who can better interpret the supposed situation 'carry off the friendly palm.'

I have expressed, then, as well as the extreme difficulties of the problem would permit, the opinion that Kant is essentially right, that no possible experiment could decide the issue between a finite distribution of bodies in space and an infinite, between a finite world-history and an infinite. The question remains: What of it?

I need hardly say that no one who deals with such a question is tempted to his task by an interest in a world's destiny. The game is finite enough so far as he is concerned, and his curiosity respecting the fate of the theatre of his brief act must be trifling. The real question that does excite the interest of reflection is that on which Kant focuses our attention. Is not the issue between these two alternative assumptions purely one of fact, and is not the fact involved unknowable? Must there not then be such a thing as an unascertainable fact, one lying beyond the reach of possible experience, in a word a *Ding-an-sich?*

It will be remembered that Kant's own answer to the question: What then? was eminently practical. He saw in the situation nothing but the occasion to point a moral; one which took at his hands the form of a command. In your judgment of what the world is, take into account as much of its distribution in space, as much of its history in time as you can. Our considerations would only modify and that but slightly, the form of the injunction. The omission from our calculation of a spatial presence leads to a constant error. The issue between a finite and an infinite history reduces so far as we can see to a question of the probable error of experiment. Our imperative would then become: Neglect no source of constant error, reduce the probable error of experiment more and more. All of which the 'laboratorius' would be in haste to acclaim, — as a commonplace.

The interest of the problem seems to me to center in the light its discussion throws upon the question: What is a fact? The answer that Kant returns to this question makes him in his own eyes an idealist; but when that answer is stated in its clearest form, when the idealism implied by it is reduced to its simplest terms, one has the conviction that the Kant of the Dialectic is an idealist of a very different stripe from the Kant of the Æsthetic and the Analytic; that furthermore it is this Kant if any who can define the meaning of a *Ding-an-sich*, and defend the right of such a concept to a place in an idealistic system.

And Kant's solution of this particular puzzle respecting an unknowable fact would seem to reduce to this: It is the very nature of a fact to be unknowable; but unknowable only in the sense that an *ideal* is unattainable. That which as a '*Gegebenes*' refusing to be found, would be a real puzzle from which the various mysticisms of a '*docta ignorantia*' might spring, becomes, when viewed as an '*Aufgegebenes*,' an ideal of method whose appearance in the discussion has the value of stating briefly the distinction between a less true and a more true experimental result, and of pointing the corollary that for every result that is 'true,' there is always definable and always ascertainable a 'truer.'

If I am right in reinterpreting the "regulative principle" in which Kant's discussion culminates to mean, 'reduce your probable error, eliminate more and more of the inexhaustible sources of constant error,' the ideal fact is the empirical result whose probable error is zero and for which a source of constant error is not definable. The interest of the antinomy we have discussed consists not in the discovery of one question of fact whose decision lies beyond the reach of possible experience, but in the demonstration which it offers that all statements of fact must retain an expression for probable error and must yield a definition of possible sources of constant error. The 'real' fact is an 'ideal' which we can define only in terms of a method that possesses a device for distinguishing between the more and less real.

In Kant's many treatments of the noumenon, there seems to me no statement of the necessity for retaining this concept so clear and so convincing as the passage of the section "Von dem

Grunde der Unterscheidung, etc." : "wo nicht ein beständiger Zirkel heraus kommen soll, das Wort Erscheinung schon eine Beziehung auf etwas anzeigt, dessen unmittelbare Vorstellung zwar sinnlich ist, was aber an sich selbst . . . etwas, d. i. ein von der Sinnlichkeit unabhängiger Gegenstand sein muss." If there is, as Jacobi suggests, a difficulty in becoming a Kantian without retaining enough of a noumenon to make our word phenomenon mean something, it is a difficulty analogous to that which would arise in maintaining that all hands are right hands. If there is a difficulty in remaining a Kantian because of this noumenon, it is owing to the mistaken inference that because a noumenon must be something different from *any* phenomenon, it must be defined without reference to a series of phenomena. Suppose one were to maintain that the method of distinguishing between the 'appearance' and 'that which appears' was one that defined and made attainable a 'real' for every 'appearing,' only that this 'real' was no less an 'appearing' that defined a 'more real' and so on *in infinitum.* There is no longer a circle but a progress, and if one defines the goal of this progress as an 'ideal,' it is none the less true that only a progress can define an 'ideal.' And it is only in the possibility of progress that one can be interested.

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